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A proposed model of the bursting process in turbulent boundary layers

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A model is proposed which attempts to explain the complete 'burst cycle'. This model views the wall streak as a sub-boundary layer, within the conventionally defined boundary layer, and the lift-up stage of bursting either as an upwelling motion of this sub-boundary layer which is similar to a local, convected separation or, equivalently, as the consequence of a vortex roll-up. 'Sweeps' are thought to represent the passage of a previous burst from further upstream. They appear either to impress on the wall streak the temporary adverse pressure gradient required to bring about its lifting or, alternatively, to provide the outer vortex which rolls up with the vortex associated with the wall streak. The model is also used to explain how the interactions between a burst and a sweep bring about (i) breakup, as well as (ii) new wall streaks further downstream.

Arguments are presented to demonstrate that the three kinds of oscillatory growth reported by Kim, Kline & Reynolds (1971) may be associated with just one type of flow structure: the stretched and lifted vortex described by Kline *et al.* (1967).

1. Introduction

Analysis of the motion pictures produced during an earlier visual study (Offen & Kline 1973, 1974) suggested that there was an interactive association between 'bursts' and 'sweeps', the dominant flow modules of the 'near-wall region' (i.e. the sublayer, buffer zone and logarithmic region).‡ These flow modules merit intensive study because together they appear to account for the majority of the turbulence production (Wallace, Eckelmann & Brodkey 1972; Willmarth & Lu 1972). The typical pattern observed was for sweeps to precede bursts, which in turn were followed by new sweeps further downstream. However, not every burst led to a sweep, nor did every sweep lead to a new burst. When these results were reported by Offen & Kline, the term 'association' was used to describe the relationship, or possible interaction, between sweeps and subsequent bursts as well as that between bursts and subsequent sweeps. The current paper presents

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‡ The term 'flow module' is taken from Morkovin (1972), who proposed it to describe "identifiable, morphologically invariant, mildly interacting, flow structures" which can frequently be observed in complex three-dimensional flow fields. Thus 'structures', 'patterns', 'events', etc., all refer to, or describe, flow modules.

a model which attempts to explain these associations, i.e. to synthesize an overall view of the quasi-cyclic events that seem to maintain the turbulence.

This search for a unifying framework for the various flow modules, which, by and large, have been studied individually by previous investigators, was motivated by two considerations. First, the repetitive nature of the flow patterns near the wall suggested a quasi-cyclic process; that is, a sequence of events which repeats in space and time, but not periodically at one place in time nor at one time in space. Second, we felt that enough information had now been collected on the behaviour of the basic flow modules as separate structures to attempt a synthesis.

The object of the tentative model outlined below is to explain why and how the two basic flow modules of the near-wall region interact (very little attention is given to the outer, or wake, region in this paper). Specifically, it is an attempt to describe the visually observed sequence of events in terms of more familiar flow phenomena, such as motions akin to local separation, motions induced by vorticity, etc. Although the motions observed during some portions of the cycle could be explained in more than one way, the models suggested here were chosen because they appear to give a coherent interpretation of the whole cycle. In fact, throughout this entire paper the reader should keep in mind both the function and the limitations of flow-visualization studies. The function of a visual study is to suggest spatial and temporal structures which can guide later studies of greater precision and dètail. Such visual studies are necessary because the complexity of three-dimensional time-dependent flows, coupled with the severe loss of information owing to long-term averaging, makes the disclosure of flow structures by one or a few conventional probes in Eulerian reference frames at best very difficult and at worst impossible. Once the structures have been disclosed, however, probes can be used to good effect to refine understanding and increase precision. Thus the results of visual studies are usually intended to be suggestive rather than ultimately conclusive (see Bradshaw (1970) for a discussion of the limitations of flow-visualization techniques). For these reasons, the results which follow need to be understood as a kind of information which is a necessary step, but which still is speculative, further, more precise studies of other kinds being required in order to reach final judgments.

The explanations presented below are provided in the hope that improved models of the basic structures will aid in the construction of computational procedures which use assumptions or data about these fundamental flow patterns (see for example Reynolds 1974). If the proposed model is correct, or even very roughly correct, it should also provide a rational basis for the design of future experiments in turbulent boundary layers. At the very least, we hope that the ideas presented here will stimulate other investigators to develop improved and experimentally verifiable models that represent the turbulence production process as a quasi-cyclic phenomenon. One apparently useful effort to construct a theory for the inner layers based on a quasi-cyclic model has already been reported.†

The model does not yet include information about the magnitudes or rates of motion of the structures; therefore, it is not predictive and, in its present form,

† See York & Abbott (1973) and Loudenback & Abbott (1973).

cannot be inserted directly into a turbulent boundary-layer computation program. Furthermore, although the model is consistent with most of the results that have been reported to date by investigators who used visual techniques, some contradictions seem to exist between interpretations of the visual data and the present model. At this time, the sources of these apparent inconsistencies are unknown, and further research is needed. These questions are discussed in detail in Offen & Kline (1973).

The following four sections contain (i) a statement of the model, (ii) a discussion of the relationship between this model and the stretched-vortex model suggested by Kline *et al.* (1967), (iii) a detailed explanation of the model and (iv) a series of comparisons between the model and relevant data.

2. The model hypothesis

Two explanatory comments are necessary before the model hypothesis can be presented. First, since the visually observed modules all move and interact in an environment of significant background fluctuations, particular features of each basic element, such as size, shape, orientation and speed, vary from one occurrence to the next (see Offen & Kline (1974, p. 237) for a more detailed discussion of this point). However, the basic character of the modules does not change between occurrences. Indeed, if it did, one could not describe or define the modules. For purposes of simplicity, the description of the model given below does not contain references to these varying influences; however, nothing in the model prevents the detailed characters of successive modules from differing as a result of interactions with the background turbulence. Second, the frame of reference used throughout the description of the model is that of an observer moving with the flow module. Therefore, since the 'lifting module' which will be introduced below is convected downstream by the mean flow, terms such as separating, back or reverse flow *only imply the existence of reverse flow relative to the local mean speed* (i.e. *if* $U = \overline{U}+u$, *then* $0 < U < \overline{U}$). *Reverse flow was never seen by a stationary observer in laboratory co-ordinates during the zero-pressure-gradient visual experiments.* The local recirculation cell, whose existence is implied by treating lift-up as a form of separation, is also convected by the mean flow, or at least by the bursting fluid. Hence, the absence of reverse flow in a stationary reference frame is not inconsistent with the kinematic requirement of reverse flow along a limiting streamline of a separating flow (Maskell 1955).

We shall arbitrarily begin the description of the quasi-cyclical model with the low-speed wall streak, which is viewed as a sub-boundary layer within the conventionally defined turbulent boundary layer. The term 'sub-boundary layer' is used here because the low-speed streak, when observed in flow-visualization studies, appears to grow in a manner which is reminiscent of the development of a conventional laminar boundary layer near the leading edge of a flat plate. Since this sub-boundary layer is of finite extent in the spanwise direction, it is a three-dimensional structure. Furthermore, as it grows, it is convected downstream by the mean flow (refer to figure 1, which is described in detail in § 4). Lift-up of the wall streak, or sub-boundary layer, may be akin to separation due to a temporary

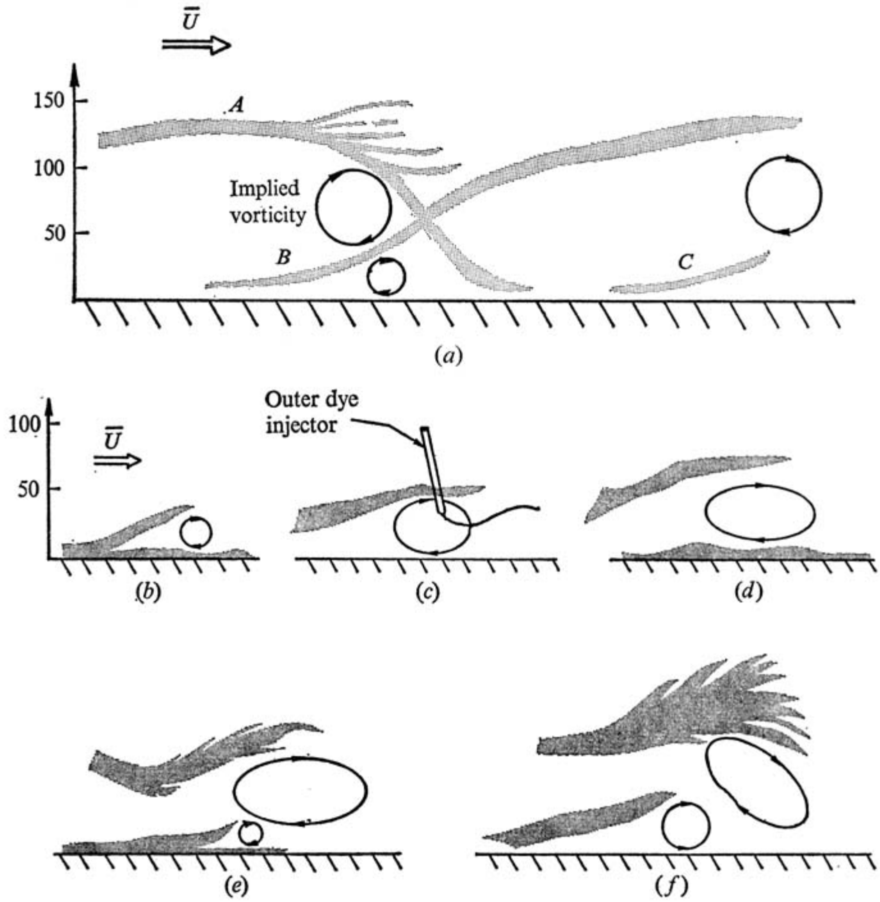


FIGURE 1. Side view of the interactions between bursting flow modules (model interpretation). (*a*) Composite, or time exposure, of dye pathlines for portions of three successive bursts (designated *A*, *B* and *C*). (*b*)–(*f*) Instantaneous views of a single burst and its interaction with another lift-up. (*b*) Lift-up and vortex associated with local convected separation bubble. (*c*) Growth stage, showing passage over an outer dye injector. (*d*) Passage over next wall streak further downstream; first burst appears as a sweep to next wall streak. (*e*) Next lift-up as first burst continues to pass overhead. (*f*) Breakup of first burst as it interacts with next lifting wall streak.

local adverse pressure gradient. However, to avoid confusion we shall refer to the structure associated with lift-up as a 'lifting module'. This module consists of the ejected fluid and a 'recirculation cell', which, as we argue later, forms below and to the side of the outgoing fluid.

The pressure oscillation required to bring about the 'lifting module' is probably imposed on the low-speed streak by the arrival of a 'sweep' near the wall. Sweeps are generally associated with a circulatory flow pattern that is characterized by *relative* backflow along the side adjacent to the wall (e.g. a transverse vortex whose vector is aligned with the vorticity due to the mean flow; see Offen & Kline 1974), and hence they impose a positive pressure gradient on the wall beneath them.

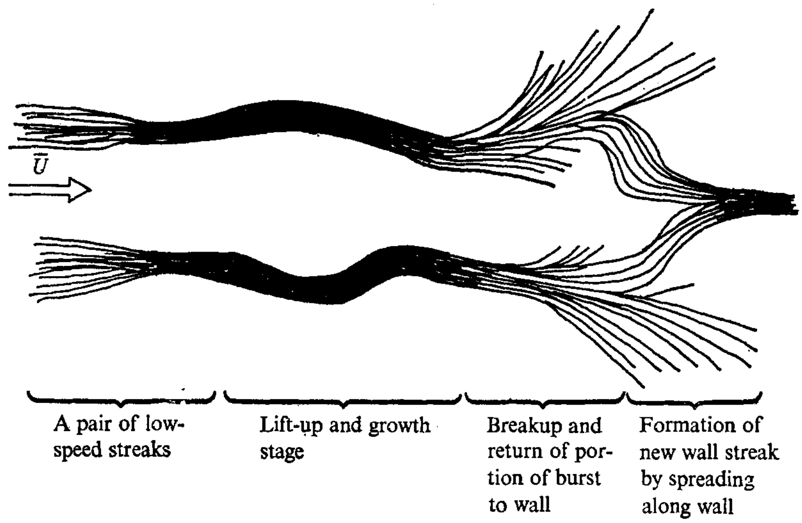


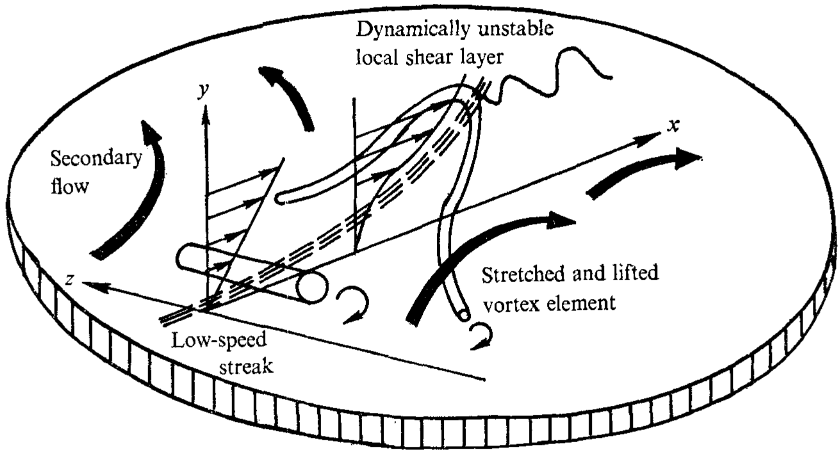
FIGURE 2. Schematic time exposure (plan view) of dye motion for two adjacent bursts during one cycle of events. The two bursts are not necessarily in phase with each other.

Some fluid from both the burst and its associated sweep returns to the wall, possibly along the downstream edge of the local, convected recirculation cell. When this fluid arrives at the wall, it spreads out sideways and is quickly retarded by the strong viscous forces near the wall. The next low-speed streaks further downstream are probably formed from such newly arrived fluid and seem to be located at spanwise positions which are approximately midway between adjacent upstream wall streaks (see figure 2).

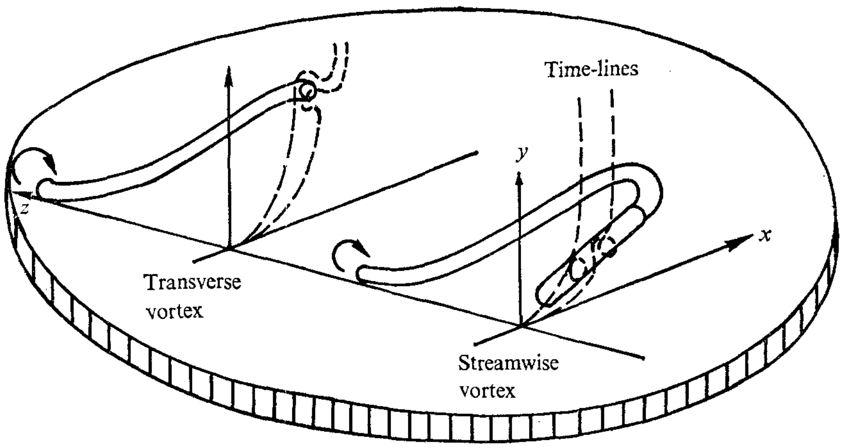
Since the origin of the wall streak is a fluid element of finite volume, the sub-boundary layer probably disappears quickly during lift-up. When this happens, the convected recirculation cell is also swept away and, therefore, new fluid will move in rapidly. The motion of the replacement fluid may be the 'cleansing sweep' which Corino & Brodkey (1969) saw near the wall at the end of the ejection.† Their suggestion that this sweep terminates the ejection stage of the burst may be another way of describing the disappearance of the convected cell.

Lift-up may also be viewed as the consequence of a vortex roll-up, or pairing, which occurs when the vortex associated with a previous burst from further upstream passes over the vortex associated with the wall streak (see § 4 and figure 1). Portions of the outer vortex would move towards the wall during such a roll-up process and hence would provide the fluid for the next streaks, as discussed earlier.

† Offen & Kline (1974) used the adjective 'cleansing' to distinguish sweeps (inward motions of high-speed fluid) which appear near the end of the ejection process from those which precede it.



(a)



(b)

FIGURE 3. (a) The mechanics of streak breakup; adapted from Kline *et al.* (1967). (b) Time-line patterns at different locations of a lifted and stretched vortex element.

3. Relationship between proposed model and stretched-vortex description.

The proposed model can be viewed, in part, as an alternative to the stretched- and lifted-vortex description given by Kline *et al.* (1967); see also figure 3 (a). The main purpose of their discussion was to justify the postulate that the vortex mechanism causes lift-up. However, it may be more useful to view the stretched vortex as a *representation* of the fluid motions associated with lift-up. When an element of fluid is lifted up and moves away from the wall, other fluid must move into the region being vacated by this burst. Since the ejected fluid, which is represented by the three-dimensional sub-boundary layer in the present model, is of finite extent in the spanwise (z) direction, replacement fluid comes from areas near the wall that are adjacent to the bursting mass in the spanwise direction as well as those ahead of it.† Furthermore, since the outward-moving, bursting fluid is further away from the wall than the inward-moving, replacement fluid, the circulatory pattern that results from this combination of motions surrounds the bursting fluid on three sides and rotates in the direction of the vorticity due to the mean flow field.‡ Time-line shapes are the same for such a circulatory pattern as they are for the stretched and lifted vortex proposed by Kline *et al.*

The discussion of the relationship between lift-up and vortex stretching in Kline *et al.* preceded observations by Kim *et al.* (1971) of the flow structure during the oscillatory growth stages; therefore Kline *et al.* did not make any attempt to relate the stretched-vortex model to one, or more, of the oscillatory growth types. One can, however, imagine that streamwise and transverse vortices merely indicate different parts of a stretched vortex. Recall that hydrogen-bubble time-lines give a picture of the flow structure only on a surface defined by the velocity vector and the wire on which the bubbles are generated. Therefore, if this plane cuts the centre, or tip, of the stretched vortex, the time-lines will show a transverse vortex (see figure 3*b*), whereas if it passes through one of the legs, the image will look like an upward-tilted streamwise vortex. From this viewpoint, the description of the oscillatory growth would serve to indicate the location of the plane of bubbles relative to the burst structure, and not to distinguish between separate kinds of bursts.

Kim *et al.* (1971) also reported that, whenever the wavy type of oscillatory growth was observed, one of the two vortex types appeared shortly thereafter. Therefore, the wavy patterns *could be* depicting pulsations in the flow which are due to the sudden lift-up of fluid slightly upstream of the observation region.

We should mention at this point that both the stretched and lifted vortex and the corresponding descriptions of the lifting flow module presented here bear a striking resemblance to some aspects of the 'horseshoe vortex' model proposed by Theodorsen (1952).

4. Clarification of the model hypothesis

The model will be clarified by discussing the following topics, in the order given: (i) the origins of the sweeps, of the wall streaks and of the bulges in the superlayer; and (ii) a plausible interaction between flow modules which explains both the very large patterns observed in flow-visualization studies as well as the chaotic ones known as breakup.

† Recall that the entire flow module is being convected downstream by the mean flow; therefore the replacement fluid from ahead (downstream) of the lift-up does not move upstream relative to the fixed laboratory frame of reference.

‡ Note that we have not said whether this circulatory pattern is rotational or irrotational; our pictures do not give us this detail. Since the bursting process appears to be a repeating sequence of vortical interactions (in time and space, not at one location in space), it is virtually impossible to follow a given vortex for more than one or two interactions. Therefore, one cannot clearly determine the origin of the vorticity in any given circulatory pattern. However, these slowly turning patterns certainly could be rotational because they are formed from fluid within the boundary layer, and this fluid does possess vorticity.

The fluid motions which cause the apparent local temporary adverse pressure gradients and which seem to serve as the origins of future wall streaks can be adequately explained by either of two equivalent kinematic descriptions: one based on the motions of elements of fluid, which are what dye streaklines make visible, and one based on the known behaviour of vortices.† These explanations will be presented below in the order just stated, with the pictorial support of figures 1 (*a*)–(*f*). (Figure 1 (*a*) is a composite view; figures 1 (*b*)–(*f*) show separately the various stages of a burst.)

According to the model, a local convected recirculation cell forms below the bursting fluid upon lift-up of a low-speed wall streak (figure 1 *b*). This flow module is a continuous three-dimensional structure located between the ejected fluid and its surroundings to the side and ahead of this lifting mass. For simplicity, the discussion will be restricted to that part of the bubble which is located just downstream of the bursting fluid and which, therefore, appears as a transverse vortex on hydrogen-bubble time-lines generated along a normal wire. As the ejection progresses, both the dye-marked fluid near the wall and the associated convected recirculation cell will move away from the proximity of the wall. In addition, the cell will grow in size. If the outer, foremost part of the slowly rotating fluid mass is far enough away from the wall when the whole structure is convected past a dye injector (e.g. one at $y^+ \simeq 100$ in our low-R_θ flow), it will be made visible as a wallward motion of the outer dye, i.e. as a sweep (see figure 1 *c*). Therefore, the sweep appears to be a portion of the flow module that described the circulatory pattern associated with a burst from further upstream. The flow along the lowest portion of the convected cell, which is closer to the wall, will be in the reverse direction when viewed by an observer travelling at the convection speed of this recirculation cell. Since, in the absence of momentum influx, fluid can only move from a region of higher to one of lower pressure, the relative reverse flow near the wall implies that the fluctuating pressure field there is temporarily characterized by a local adverse pressure gradient. When this structure passes over a low-speed wall streak, the correct conditions will exist for another lift-up; i.e. for another lifting module to develop from a sub-boundary layer (refer to figures 1 *d*, *e*).

During an earlier visual study (Offen & Kline 1974), we observed that sweeps usually come down to the wall just ahead of the bursting wall fluid, spread out there and slow down rapidly. Therefore we are led to suggest that this incoming fluid, which probably arrives at the wall near the downstream end of the local convected recirculation cell, may be the origin of the next low-speed streak.

The alternative explanation for lift-up relies on vorticity concepts.‡ Arguments in the previous section about the secondary flow patterns associated with lift-up strongly imply that bursting fluid is surrounded by a stretched and lifted

† The ideas to be presented here seem to have been presaged by Laufer & Badri Narayanan (1971), who suggested that "the turbulent 'bulges' moving away from the wall into the potential field and caused by the vortex breakdown (bursts) near the wall produce a pressure field that gives rise to an instability of the sub-layer (lateral periodicity), thereby initiating the production sequence".

‡ We emphasize that 'vortex' modules do not explain a different flow structure than do velocity-field modules; the two merely give alternative descriptions of the same field. One or the other may be more useful for a given purpose.

vortex. The visual results also indicate that sweeps represent vortical motions, and therefore it is natural to seek a relationship between bursts and sweeps based on vortex interactions. Although the pertinent flow modules will frequently be described in terms of vortices in the next few paragraphs, the reader should be reminded at this point that the vortical structures observed during the visual studies were not like the typical bathtub vortex with a large circulation; instead, they were very slowly rotating structures caused by slight velocity differences at two adjacent locations. They often did not even make a complete rotation before they interacted with another flow module and lost their coherence as a single, unique structure. This visual observation is consistent with the statement by Tennekes & Lumley (1972), based on dimensional arguments, that the "large eddies lose a significant fraction of their kinetic energy, $\frac{1}{2}\overline{u^2}$, within one 'turnover' time".

Results obtained by Winant (1973) for a free shear layer behind a splitter plate suggest the nature of the vortical interactions between bursts and sweeps. The vortices shed from the trailing edge of the plate are, of course, all oriented in the same direction, and Winant found that they interact in pairs by rolling around each other. The result of any given interaction is a single vortex of about twice the scale of the former vortices. Moreover, Winant suggested that this vortex pairing process may be the dominant feature in the shear layer since, by using the model, he could compute a fluctuation energy profile for the streamwise velocity which agreed favourably with the measured profile.

Now, in boundary-layer flow, a stretched vortex that has already been lifted away from the wall will rotate and translate faster than the vortex associated with a streak which is still attached to the wall, but both will turn in the same direction.† Therefore, when a lifted vortex from a previous burst further upstream is convected downstream to a location above a low-speed streak (as shown in figures 1 *d*, *e*), pairing becomes possible. The faster, lifted vortex induces a rapid outward motion of the wall-dominated vortex with a consequent large acceleraton of the associated bursting fluid. However, conversely, the outer vortex is induced to move only gradually towards the wall by the slower, wall-dominated vortex.

Note that the scaling arguments which form the basis of the log law imply that the size of typical flow modules in the logarithmic region is proportional to their distance from the wall. The concept of growth by pairing presented above is consistent with these scaling arguments.

The bulges in the superlayer (Fiedler & Head 1966; Kaplan & Laufer 1968; Kovasznay, Kibens & Blackwelder 1970) also seem to be related to the interactions which take place where the inner and outer layers merge. The vortical structures that dominate the flow in this segment of the boundary layer appear to grow either by pairing once more with another vortical structure or by inducing the less organized surrounding fluid to circulate. Since these vortical structures, representing old bursts from further upstream, occupy most of the space in the boundary layer, the opportunity for growth by fusion, or pairing,

† The vorticity due to the non-uniform velocity profile within the streak, or sub-boundary layer, can be viewed as a vortex from a convected reference frame.

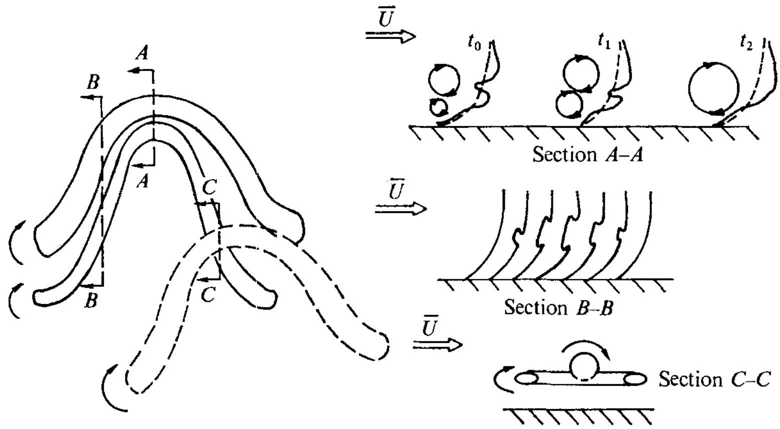


FIGURE 4. Plan view of the interactions for two possible pairings of the stretched vortices associated with bursting flow modules (model interpretation). The upper pair (both solid lines) are aligned with each other and produce large-scale structures (sections A–A and B–B). The lower pair (thin and dashed line) are not aligned and their interaction (section C–C) may appear as 'breakup' in visualization studies. Section A–A shows bubble lines at three successive times plus associated vorticity and mean profile (dashed line). Completion of roll-up at t_2. Section B–B shows large streamwise vortex in instantaneous view of bubble lines after roll-up (time $= t_2$). Section C–C is vortex representation of interaction between non-aligned flow modules. Circular vortex is same as dashed one in plan view.

certainly exists. Such an expanded flow module may very well be the large-scale structure whose upper part is observed as a bulge in the superlayer. However, our observations on this point are restricted to low R_θ and are incomplete even for that case.

The roll-up process will occur in the manner described in the preceding paragraphs only if the lifted vortex from the previous burst is aligned with the vortex associated with the wall streak (figure 4). Although one would not expect this to be the common vortex pattern, roll-up will be described for aligned vortices first because that situation is easier to comprehend. When the lifted and the streak-associated vortices are aligned, their heads will be able to interact as if they were both segments of a two-dimensional structure such as the ones in Winant's (1973) experiment. When the heads of the two stretched vortices are aligned, so are their legs; therefore they too will roll up to form a large flow module. Such interactions apparently do occur, because the large-scale patterns depicted in sections A–A and B–B on figure 4 are occasionally observed near the wall in the bubble pictures. When these patterns depict the roll-up of the tips of two stretched vortices, the resulting large transverse vortex is clearly evident (e.g. figure 4, section A–A). However, the patterns which may be due to the roll-up of two vortex legs can only be deduced from the other typical large-scale structure that is observed in the time-lines. As depicted in section B–B in figure 4, these appear as a large element of fluid (the zone between the kinks in the time-lines) moving with a speed different from that of the surrounding fluid and separated from it by two thin layers (the kinks). These layers, which represent regions of velocity jumps in both the streamwise and the spanwise (normal to the page) directions, are approximately parallel to the wall, but located at different distances from the wall. It is difficult to ascertain the spanwise velocity components of these kinked portions of the time-lines from the view shown here, but if the two lines of kinks do move in opposing spanwise directions, then such patterns could be depicting a cross-section of a slowly rotating, large, streamwise vortex whose translational speed differs from the local mean speed. A circulatory motion of this kind might represent the result of vortex roll-up along the two sides of a pair of aligned stretched vortices.

However, good alignment between two flow modules is rare in a turbulent environment. Therefore the typical interaction between an upper and a lower vortex ought to involve structures that are not aligned (also shown in figure 4) and hence produce complex, three-dimensional motions instead of nicely defined structures. These motions may be the ones that are made visible as chaotic patterns by the time-lines and that have been called 'breakup' by Kim *et al.* (1971).

Evidence to support this explanation of breakup comes from combining the visual observations reported by Kim *et al.* with the description given earlier in this section to explain the origin of wall streaks. Kim *et al.* noted that some of the dye-marked wall fluid returned to the wall during breakup. We have already suggested that the inflow of fluid towards the wall appears to be part of the interaction between a previous burst from further upstream and the local burst, and that this interaction can be described as a roll-up of the vortical structures associated with each burst. Therefore, since the inflow is treated as a consequence of both breakup and the pairing of the two vortices, it seems reasonable to believe that breakup is closely related to this interaction between the two vortices.

Additional evidence comes from visual analysis of the flow patterns in the region surrounding breakups.† These patterns revealed that at least 80 % of the bursts broke up immediately after an interaction with another vortical structure. The interpretation just given for the breakup process could explain this observation.

A consequence of viewing breakup as an interaction between non-aligned vortices is to make this process responsible for redistributing a significant fraction of the fluctuation energy from the u velocity to the other two components (the remainder of the energy redistribution probably occurs on account of the vortex stretching during lift-up). When the stronger, streamwise fluctuations near the centre of a previous lift-up interact with the weaker, skewed fluctuations along the side of an incipient lift-up during a vortex roll-up, energy is transferred from the x-direction motions to the y- and z-direction oscillations. The existence of such isotropy-producing tendencies has already been inferred by previous investigators from the equations of motion and from experimentally determined velocity fluctuation profiles.‡

† Unreported visual experiments at Stanford.

‡ See, for example, Townsend (1956), Bradshaw (1971) or Tennekes & Lumley (1972) for discussions of the meanings of the various terms in the equations of motion (Navier–Stokes equations). The experimental evidence comes from studies such as those by Klebanoff (1956) and Bradshaw (1967).

5. Consistency between the model and related data

The model described above, and particularly that portion of the model which concerns itself with the growth and lift-up of the low-speed wall streak, can be used to explain the *trends* observed in nearly all the relevant data known to us. The model proposed here cannot definitively explain, confirm or refute data such as the burst-rate observations of Rao, Narasimha & Badri Narayanan (1971) or Meek (1972), because it does not give any quantitative information; it merely suggests reasons for the sequence of events observed in the visual experiments and for the effects of varying boundary conditions on these events. The relationships between the proposed model and the results obtained by several investigators when they varied some of these boundary conditions will be explored in this section.

Although the present model gives a more precise and comprehensive kinematic description of the visual patterns observed in turbulent boundary layers than the 'wall-layer hypothesis' proposed by Runstadler, Kline & Reynolds (1963), it does not conflict in any way with the basic tenets of that hypothesis. Therefore the arguments which Runstadler *et al.* presented to show consistency between their hypothesis and numerous sources of related data also apply to the present model and will not be repeated here. The following discussion will concentrate on relationships with more recent data.

We should first recall that the model emphasizes the interactions in the logarithmic or mixing region of the turbulent boundary layer. This stress on the interactions in the mixing region is a characteristic of the model irrespective of whether the sequence of events is described in terms of velocities or vortices. The visual data of Corino & Brodkey (1969), the measurements of Rao *et al.* (1971) and the collection of visual data accumulated at Stanford (Runstadler *et al.* 1963; Kline *et al.* 1967; Kim *et al.* 1971; Offen & Kline 1973) all indicate that the turbulence processes near the wall are due to nearby interactions; they are not driven *directly* by the large-scale motions which are observed as 'bulges' in the super-layer.†

Laufer & Badri Narayanan (1971) observed that the wall shear stress under a constant-pressure turbulent boundary layer intermittently drops to a low value. They measured the average time between these stress reductions and found that it was similar to the average burst period reported by Rao *et al.* (1971), when both times were normalized using the same outer variables. Therefore they suggested that the temporary wall-stress deficits correspond to one stage of the burst cycle. The proposed model also implies that the wall shear will be greatly reduced once during each cycle, namely, during lift-up. Of course, any model that conforms to the visual data must allow the wall shear to become negligible during lift-up;‡ therefore, although the results of Laufer & Badri Narayanan

† Although these big 'eddies' are a source of energy for the interacting structures closer to the wall, they do not seem to regulate directly the activity very near the wall.

‡ Kim *et al.* (1971) showed that bursting was associated with instantaneous velocity profiles which were inflexional and exhibited a very low dU/dy, and hence low shear stress, at the wall.

cannot be used to verify the proposed model, they were included in this discussion to show that they are consistent.

The pressure-gradient data of Schraub & Kline (1965) are perhaps the most important results with which to compare the model. They measured the burst rate as a function of pressure distribution for both adverse and favourable gradients. Since their pressure distributions were non-equilibrium ones, the effects of 'history' were also observed. If the present model is correct, one would expect a favourable pressure gradient to reduce the burst rate, because it acts in opposition to the local adverse pressure gradient and, therefore, hinders the development of the lifting module from the sub-boundary layer. Conversely, an adverse pressure gradient should increase the burst rate by supplementing the temporary local adverse pressure gradient and hence stimulating the development of the lifting module. These trends for positive and negative pressure gradients are monotonically observed in the Schraub & Kline data.

Furthermore, since the hypothesis states that a lift-up does not occur unless a sweep arrives near the wall from further upstream, and since this sweep is dependent upon a previous burst, one would expect the local burst rate to depend, at least weakly, upon the upstream pressure distribution too. For example, if the magnitude of the (mean) adverse pressure gradient is larger at a nearby upstream location X_1 than it is at the point of observation X_2, the burst rate will be larger at X_1 than at X_2. Since the burst rate is essentially equal to the sweep-generation rate, more sweeps will arrive at X_2 than one would expect on the basis solely of the pressure gradient there. Therefore, the local burst rate should be somewhat larger than what one would predict for a constant pressure gradient. Again, the results for burst rate *vs.* pressure gradient of Schraub & Kline seem to bear this out. This line of rationalization is also consistent with a whole body of data reported at the 1968 *Conference on Computing Turbulent Boundary Layers* (Kline *et al.* 1969). Results contained in this volume show first that the wall layers adjust more rapidly to changes in the applied pressure gradient than do the layers which are further from the wall, and second that history effects exist but are generally not very strong. Both these ideas are consistent with the explanation based on the model above.

During the presentation of the model, we advanced the hypothesis that the bulges in the superlayer may be the consequence of vortex pairing between the vortices associated with two (or four) bursts. Schraub & Kline (1965) also suggested that the bulges are the same as, or are at least formed from, the ejected wall streaks. They measured the streamwise velocity of the ejected wall streaks as a function of distance from the wall and found that the dye-marked wall fluid travels downstream with a speed $U \simeq 0{\cdot}8U_\infty$. Klebanoff (1956) reported approximately the same value for the convective velocity of the eddies in the outer flow. Schraub & Kline interpreted this agreement between the two sets of velocities as evidence that the outer eddies are closely related to the bursts.

We should remind the reader that our data are for a flow at relatively low R_θ; at higher R_θ one may observe additional 'pairing'. Thus the basic structure should be the same at high and low Reynolds number, but the characteristic length and time scales ought to change gradually with R_θ. This kind of difference between low and high Reynolds number flows would be consistent with known monotonic and continuous variations as a function of R_θ of the ratio of inner to outer scales, of the shear velocity and of the Stanton number. As will be mentioned at the end of this section, the existence of additional 'pairing' would also make our model agree with the flow structures observed by Falco (1974) at high R_θ.

Since the present model views breakup as part of the interaction between a previous lift-up and an incipient one along the wall, i.e. as involving the birth of one lift-up and the death of another, the distance between similar locations on successive wall streaks should be approximately equal to the distance from lift-up to breakup. The distance between similar locations on successive 'events' has been estimated to be $\Delta x^+ = 1000$–1500 (see Offen & Kline 1973). On the basis of measured trajectories of many bursts dye-marked at the wall, Schraub & Kline (1965) found an average distance $\Delta x^+ \simeq 1300$ between lift-up and breakup. Thus, the distance between lift-up and interaction with the next wall streak inferred from data obtained by the present authors is similar to the distance between lift-up and breakup inferred from data of Schraub & Kline. The postulate that breakup is a consequence of the interaction between one burst and the next one further downstream also gains support from the fact that the average time between lift-up and breakup (Schraub & Kline 1965) agrees to within 10 % with the average time between bursts (Kim *et al.* 1971) for the same flow conditions.

The belief that portions of two adjacent bursts combine to produce a low-speed streak further downstream and approximately midway between the two receives support from the experimental evidence of a mean spanwise spacing between adjacent wall streaks with a measurable, although large, standard deviation (Runstadler *et al.* 1963; Kline *et al.* 1967; Gupta, Laufer & Kaplan 1971). The creation of new streaks about half-way between their non-uniformly spaced 'parent streaks' would drive the average spanwise spacing between adjacent streaks towards a constant mean value (in the absence of other, secondary fluctuations). This is illustrated by the simplified sketch in figure 5, which is not reproduced from a flow-visualization experiment and should only be taken as suggestive. If the process of wall-streak formation were not regenerative, or cyclical, the streaks would probably be distributed in an entirely random fashion. The mean streak spacing might be as large as the half-width of the test section, and the standard deviation would then equal the mean divided by $\sqrt{3}$. Note that the arguments presented in this paragraph are valid irrespective of whether the low-speed streaks are generated in accordance with the proposed model or with a model based on pairs of counter-rotating streamwise vortices (Bakewell & Lumley 1967), which of necessity are separated by alternate zones of inward motions (supposedly the source of the low-speed streaks) and outward motions (the bursts). In both interpretations upstream bursts are the source of new streaks.

Andersen, Kays & Moffat (1972) have shown that transpiration affects the turbulent boundary layer in much the same way as pressure gradients do; that is, the effects of blowing on the mean and fluctuating profiles are the same as the effects of adverse pressure gradients on these variables and, conversely, the

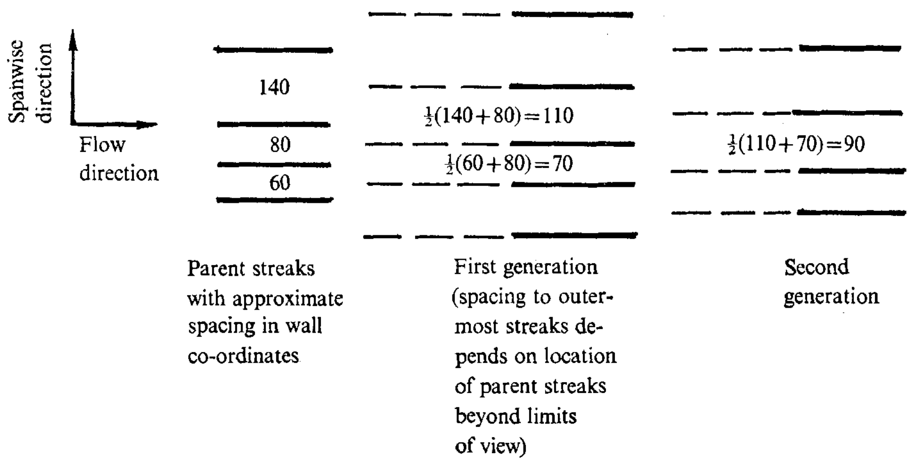


FIGURE 5. Schematic representation of streak spacing for successive generations of bursts.

effects of sucking are similar to those of favourable pressure gradients. It is also well known that boundary-layer suction inhibits separation, whereas blowing stimulates it. Therefore these results are consistent with the idea that lift-up is akin to local relative boundary-layer separation.

Bremhorst & Walker (1973) measured the cospectrum of the u and v velocities, as shown in figure 6. Inspection of this plot reveals a frequency range near the lower end of the scale where the cospectrum is slightly positive, but only for measurements taken in $7 \leqslant y^+ \leqslant 21$, i.e. near the wall. The authors state that this frequency range corresponds to the average time between bursts. These results suggest that a flow module which contains a region between $y^+ = 7$ and 21 where retarded fluid ($u < 0$) moves towards the wall ($v < 0$) passes a fixed probe once during each burst cycle. The local recirculation cell associated with bursts by the present model satisfies such a condition ($uv > 0$) and, therefore, could be the cause of this feature of the cospectrum.

Lu & Willmarth (1973) found equal average times between periods of *large* $|uv|$ due to the combination $u < 0, v > 0$ and periods of *large* $|uv|$ due to the combination $u > 0, v < 0$. The first set of velocities corresponds to the standard interpretation of bursts (outward migration of retarded fluid) and the second set corresponds to the one for sweeps (inward motion of accelerated fluid). The proposed model is consistent with these results because it requires a sweep to initiate a burst and a new burst to generate another sweep.

The proposed model is also consistent with the pressure data obtained by Tu & Willmarth (1966) along the wall under a flat-plate tubulent boundary layer. They found a slight re-rise in the pressure autocorrelation function at a non-dimensional time which corresponds to the average burst period. This means that high- and/or low-pressure pulsations occur repetitively at a rate equal to the average burst rate; in other words, a high and/or low pressure passes the sensor once during each burst cycle. If the present model is correct, the re-rise is due to the passage of both a high- and a low-pressure zone; high pressure correponds to

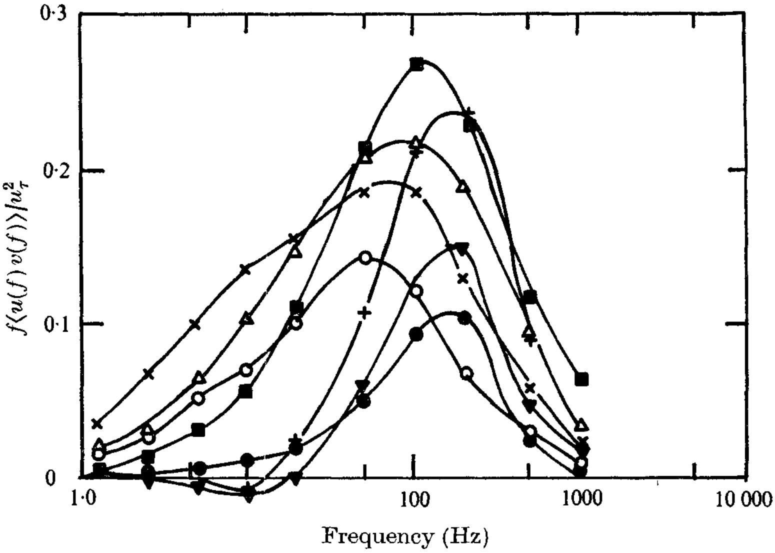


FIGURE 6. Cospectrum of u and v (from Bremhorst & Walker 1973).

	●	▼	+	■	△	×	○
y^+	6·46	8·6	12·9	21·5	64·6	215	646

the arrival of the sweep at the wall while low pressure corresponds to the local recirculation cell.

Nychas, Hershey & Brodkey (1973) also found that the bursting process can be described as a sequence of deterministic events. Although their visualization technique differed significantly from the ones used by the present authors, there does not appear to be any substantial disagreement between their observations and the proposed model. There is a difference in the explanation of the formation of transverse vortices along the high-shear zone between high- and low-speed fluid. Nychas *et al.* suggested that these flow modules are the result of a Helmholtz instability due to inflexions in the velocity profile within the high-shear zone. Kline & Runstadler (1959) and Kim *et al.* (1971) suggested similar ideas. Our model, on the other hand, postulates that the vortex *represents* the relative motions of the burst and the replacement fluid. The cause of the lift-up is, therefore, also the cause of the vortex, and we have suggested that the lift-up is due to the temporary passage of a relative adverse pressure gradient over a wall streak (also suggested by Nychas *et al.* as a possible explanation for the ejections they observed).

This difference in the interpretations cannot be unambiguously resolved from the data contained in Nychas *et al.* and in Offen & Kline (1973), because visual observations can only show relative locations and times of events; they cannot demonstrate the cause of any event within a sequence. However, we do believe that it is more reasonable to view the ejection, and hence the associated transverse vortex, as the response of the wall streak, or sub-boundary layer, to an adverse pressure gradient than as the response of sweeps, whose motions have to be treated as travelling waves, to a random perturbation in the presence of shear. We remind the reader that a transverse vortex always depicts a structure characterized by relatively fast flow along one side and relatively slow flow along the other side, and that this relationship applies at the inception of the vortex irrespective of its cause. The ejection/replacement flow module described earlier is such a structure.

Nychas *et al.* proposed an explanation for the transverse vortices different from that of Kim *et al.* (1971), in part because they observed a much greater ratio of transverse to streamwise vortices than was reported by Kim *et al.* Since they suggested that the disagreement over the origin of transverse vortices is due to an apparent difference between the two visualization techniques, an explanation is in order. The depth of view in their photographs is $\Delta z^+ = 150$–200, and therefore, when they observed a burst, they generally saw the entire structure. Since a transverse vortex depicts motions in the plane perpendicular to the line of sight of the camera, it is more readily observed than a streamwise vortex, which portrays motions that are primarily parallel to the line of sight. Therefore, if each ejection is surrounded by a vortical structure and hence represented by both types of vortices as depicted in figure 3, the pictures of Nychas *et al.* ought to have been dominated by a transverse vortex each time they saw an ejection. Kim *et al.*, on the other hand, could only see those motions which coincided with the surface formed by a sequence of bubble time-lines. Since the 'legs' of the stretched and lifted vortex shown in figure 3 span a greater z-distance than the 'head' does, the time-lines are more likely to cut through one of the legs and be deformed into a streamwise vortex than to cut through the head and appear as a transverse vortex. Therefore, Kim *et al.* should have found more streamwise vortices than transverse ones. It should be noted, in conclusion, that the 'two-layer velocity effect' described by Nychas *et al.* may in fact depict a streamwise vortex.

Despite our arguments in the preceding paragraph, we should mention that the difference between a wave (instability) representation and a coherent-structure (eddy or vortex) representation may not be significant in this special instance. We make this statement for the following reason. Lahey & Kline (1971) have shown that the available two-point space–time correlation data for shear flows can be reproduced by mathematically representing the Cartesian velocity components as the sum of a large pink-noise component and a wave component. The wave portion of this representation requires (i) a stochastic phase coefficient and (ii) a 'jitter' on the basic wavenumber. Travelling waves with these properties can be 'packet-like' and hence little, if any, different from coherent eddies. What seems important in the turbulence production process, and where there seems to be general agreement, is in the quasi-cyclic nature of the processes. The remaining disagreements may be no more than different descriptions of the same phenomena.

The proposed model also agrees well with the visual observations of Falco (1974), at least those from his moderately low Reynolds number flow. He observed a structure in the outer, intermittent region of the turbulent boundary layer whose shape and circulatory pattern were similar to those reported here for bursts near the wall. According to Falco these flow modules first appear near the wall, but they can be followed across the entire boundary layer at low R_θ (Falco's high-R_θ pictures can be reconciled with the model suggested herein if one accepts the idea of repeated 'pairing' of the vortices associated with bursts). Furthermore, the structures occasionally return to the wall from the middle of the boundary layer, rolling over as they move inwards. At other times two, or more, of these flow modules coalesce to form a larger structure. Although we do not endorse all of Falco's arguments (particularly with respect to the differences between flow structures at low and high R_θ), these observations are consistent with the ideas in the present paper.

6. Summary

This paper is an attempt to synthesize most of the previous interpretations of the structure of the turbulent boundary layer which are based on visual data. Arguments are presented for the idea that streamwise vortices, transverse vortices and wavy growth, the three possible modes of oscillatory growth during a burst (Kim *et al.* 1971), merely represent different views of, or cuts through, the stretched and lifted vortex that Kline *et al.* (1967) associated with bursting. The transverse vortices seen using hydrogen-bubble time-lines from a normal wire may represent stretched and lifted vortices whose heads, or leading parts, pass through the plane of the time-lines, whereas the streamwise vortices may represent those whose legs, or sides, pass through this surface. Since wavy growth is always followed by either a streamwise or a transverse vortex, this pattern may represent internal waves which propagate down from a lifted vortex that is still further upstream.

A kinematic description of the relationships between bursts and sweeps is tentatively suggested in an effort to explain in greater detail the quasi-cyclic events near the wall which appear to maintain the turbulence. In the proposed model a low-speed wall streak is viewed as a sub-boundary layer within the conventionally defined turbulent boundary layer. Lift-up of this inner layer may be akin to convected separation brought about by a temporary local adverse pressure gradient. The circulatory flow in a burst, or lifted wall streak, is such that it could impose an adverse pressure gradient on the wall as it passes over a newly forming wall streak. Moreover, the older burst (i.e. the upstream one) would be made visible as a sweep by visualization devices located in the logarithmic region near the origin of the new (downstream) burst. Some fluid from both bursts returns to the wall, where it spreads out sideways, is quickly retarded, and may be the source of new low-speed streaks further downstream.

Since bursts are associated with vorticity, generally in the direction of the mean vorticity, the lift-up and return-flow process could also be explained by roll-up of the vortices of two bursts as the older one passes over the nascent one. These vortices are usually not aligned with each other; therefore the resulting motions will be complex and may very well represent the breakup stage of a burst described in Kim *et al.* (1971).

Although the proposed model does not yield quantitative data, it is consistent with the trends in all the relevant data known to us.

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